THE MYTH OF MODULARITY IN RULE-BASED SYSTEMS FOR REASONING WITH UNCERTAINTY

David E. Heckerman and Eric J. Horvitz

Medical Computer Science Group, Knowledge Systems Laboratory
Medical School Office Building, Room 715, Stanford, California 94305

In this paper, we examine the concept of modularity, an often-cited advantage of the rule-based representation methodology. We argue that the notion of modularity consists of two distinct concepts: which we call semantic modularity and syntactic modularity. We argue that when reasoning under uncertainty, it is reasonable to regard the rule-based approach as both syntactically and semantically modular. However, we argue that in the case of plausible reasoning, only semantically modular but not necessarily syntactically modular. To illustrate this point, we examine a particular approach for managing uncertainty in rule-based systems called MYCN. We show how to define the concept of semantic modularity with respect to the certainty factor model and discuss logical consequences of the definition. We show that the assumption of semantic modularity imposes strong restrictions on rules in a knowledge base. We argue that such restrictions are rarely valid in practical applications. Finally, we suggest how the concept of semantic modularity can be relaxed in a manner that makes it appropriate for plausible reasoning.

1. INTRODUCTION

Researchers in the artificial intelligence community have concentrated their efforts on deductive problem-solving methods. In doing so, they have developed numerous representations for representing knowledge. One methodology that has been used frequently to build expert systems is the rule-based representation framework. In rule-based systems, knowledge is represented as rules of the form "If A AND B THEN C", where A and B are logical propositions.

An often-cited advantage of the rule-based approach is that rules can be added or deleted from a knowledge base without the need to modify other rules [1]. This property is called modularity. To our knowledge, the concept of modularity has never been formally defined. Nevertheless, modularity has been informally described in some detail. For example, the following two paragraphs are taken from a discussion of modularity by Davis [1]:

"We regard the modularity of a program as the degree of separation of its functional units into replaceable pieces. A program is highly modular if any functional unit can be changed by recompilation with no unintended change to other functional units. This program modularity is inversely related to the strength of coupling between functional units.

The modularity of programs written in pure production systems arises from the important fact that the net rule to be invoked is determined solely by the common to the this base, and no rule is ever called directly. Thus the addition (or deletion) of a rule does not require the modification of any other rule to prove, nor or delete a rule to it. We might demonstrate this by separately accessing rules from a PC. Production systems do sometimes call subgoals, which, in the absence of a mechanism for controlling subgoals, can require modification of other rules of the code to ensure that the procedure is invoked, while removing an arbitrary procedure from the same program will generally create trouble in the above operation and in other directions of modularity, it seems that when different notions of modularity are defined without apparent distinction. One notion is that rules can be added or deleted from a knowledge base without altering the truth or validity of other rules in the system. The other notion is that rules can be added or deleted from a knowledge base without modifying the syntax of other rules; the inference process can continue in spite of such additions or deletions. We will call the former notion semantically modular and the latter syntactically modular.

By design, rules are semantically modular. However, since the rule-based representation scheme may have emerged from the recognition that logical rules are modular on the semantic sense. As investigators have begun to tackle real-world problems such as mineral exploration, medical diagnosis, and financial management, methods for reasoning under uncertainty or plausible reasoning have received increasing attention. Popular AI approaches that have been developed for managing uncertainty include extensions of the production rule methodology. In these methodologies, a particular rule which represents the degree of association, in some sense, between the antecedent and the consequent of the rule. In such approaches, the notions of semantic and syntactic modularity have been carried over from deductive systems. This is of both semantic and syntactic modularity have been added to rules in plausible reasoning systems. It seems appropriate to attribute the property of semantic modularity to rules in plausible reasoning systems. However, in these systems, non-canonical rules can be added and deleted without the need to modify the system of other rules. However, in this paper, we argue that it is inappropriate to carry over the notion of semantic modularity from deductive systems and to apply it to systems which must manage uncertainty. We shall see that fundamental difference between logical and plausible reasoning results in the breakdown of the assumption of logical modularity in rule-based systems which reason under uncertainty.

To demonstrate that this is incoherent, we consider the property of semantic modularity in plausible reasoning systems, we will examine a canonical rule-based method for reasoning under uncertainty; the MYCN certainty factor model [2]. We will first present a formal definition of semantic modularity with respect to the CF model. We will then discuss several implications of semantic modularity and argue that these implications cannot be asserted to most practical applications. Also in this paper, we will discuss a methodology for relaxing the assumptions of semantic modularity to accommodate plausible reasoning.

We should emphasize that we are not the first to notice problems with the assumptions of modularity in rule-based systems which reason under uncertainty. For example, in the choice theory of their book on rule-based expert systems [1], Buchanan and Shortliffe state that many of MYCN's rules do not have the property that we termed semantic modularity. However, in our knowledge, there have been no attempts to formally define the concept of semantic modularity nor have there been attempts to incorporate the concepts of semantics and
Conditional independence

The first consequence we discuss concerns a common situation where several pieces of evidence bear on a single hypothesis. This is shown below:

\[ E_1 \quad E_2 \quad H \quad E_3 \]

Let \( \mathcal{E} \) denote the set \( \{E_1, E_2, \ldots, E_n\} \). Consider a single item of evidence \( E_i \) in \( \mathcal{E} \) and let \( a \) be any subset of \( \mathcal{E} \) which does not include \( E_i \). In this situation, it can be proved that the modularity property (1) holds if and only if

\[
P(E_i \mid a, a') = P(E_i \mid a') \quad \text{and} \quad P(E_i \mid a, a') = P(E_i \mid a')
\]

(2)

Equation (1) says that the belief in \( E_i \) does not depend on the knowledge that is true when \( H \) is definitely true or definitely false. When (3) holds, it is said that evidence is conditionally independent given \( H \) and its negation.

Let us consider this correspondence between conditional independence and the modularity property in the context of the arm problem above. In the example, it is a simple matter to see why the modularity property is violated; draws are done without replacement, making evidence conditionally dependent. For example,

\[
P(\text{red draw black} \mid H, \text{list draw black}) = 0
\]

and

\[
P(\text{red draw b} \mid \text{list draw w}) = 1/2.
\]

Clearly, condition (3) is not satisfied in the example; this is consistent with our previous observation that modularity does not hold.

The arm problem can be modified such that the modularity property is satisfied. If draws are done with replacement, the conditional independence condition (3) is satisfied. In particular, for any \( i \):

\[
P(W_1 \mid w_2) = P(W_1) = 1/3
\]

and

\[
P(W_1 \cap W_2) = P(W_1) \cdot P(W_2) = 1/9.
\]

where \( W \) denotes the draws of a white ball and \( w \) denotes draws made prior to \( W \). Since (3) is satisfied, the modularity property holds and we can compute \( C(H_i \mid W) \), a function of only two arguments, using relation (2):

\[
C(H_i \mid W) = 1/(1/3 \cdot 1/9) = 5
\]

\[
C(H_{i+2} \mid W) = 5 \cdot 1 \cdot 1 = 5.
\]

Other binary factors relevant to the problem can be calculated in a similar fashion.

Unfortunately, semantic modularity only holds in extremely simple situations like the one above. Any small increase in the complexity of the problem will result in the loss of modularity. For example, support on an individual is given one of three arms:

\[
H_1 \quad H_2 \quad H_3
\]

Making draws with replacement, evidence is conditionally independent given each of the hypotheses \( H_1, H_2, \) and \( H_3 \). However, evidence is no longer conditionally independent given the negation of any hypothesis. For example, if each hypothesis is equally likely before any draw, the initial probability of drawing a black ball given \( H_1 \)

\[
p(B \mid H_1) = \frac{p(B \mid H_2) + p(B \mid H_3)}{2}
\]

Equation (1) says that the belief in \( E_i \) does not depend on the knowledge that is true when \( H \) is definitely true or definitely false. When (3) holds, it is said that evidence is conditionally independent given \( H \) and its negation.

Let us consider this correspondence between conditional independence and the modularity property in the context of the arm problem above. In the example, it is a simple matter to see why the modularity property is violated; draws are done without replacement, making evidence conditionally dependent. For example,

\[
P(\text{red draw black} \mid H, \text{list draw black}) = 0
\]

and

\[
P(\text{red draw b} \mid \text{list draw w}) = 1/2.
\]

Clearly, condition (3) is not satisfied in the example; this is consistent with our previous observation that modularity does not hold.

The arm problem can be modified such that the modularity property is satisfied. If draws are done with replacement, the conditional independence condition (3) is satisfied. In particular, for any \( i \):

\[
P(W_1 \mid w_2) = P(W_1) = 1/3
\]

and

\[
P(W_1 \cap W_2) = P(W_1) \cdot P(W_2) = 1/9.
\]

where \( W \) denotes the draws of a white ball and \( w \) denotes draws made prior to \( W \). Since (3) is satisfied, the modularity property holds and we can compute \( C(H_i \mid W) \), a function of only two arguments, using relation (2):

\[
C(H_i \mid W) = 1/(1/3 \cdot 1/9) = 5
\]

\[
C(H_{i+2} \mid W) = 5 \cdot 1 \cdot 1 = 5.
\]

Other binary factors relevant to the problem can be calculated in a similar fashion.

Unfortunately, semantic modularity only holds in extremely simple situations like the one above. Any small increase in the complexity of the problem will result in the loss of modularity. For example, support on an individual is given one of three arms:

\[
H_1 \quad H_2 \quad H_3
\]

Making draws with replacement, evidence is conditionally independent given each of the hypotheses \( H_1, H_2, \) and \( H_3 \). However, evidence is no longer conditionally independent given the negation of any hypothesis. For example, if each hypothesis is equally likely before any draw, the initial probability of drawing a black ball given \( H_1 \)

\[
p(B \mid H_1) = \frac{p(B \mid H_2) + p(B \mid H_3)}{2}
\]

However, if a white ball is drawn, \( H_2 \) is ruled out and the probability of drawing a black ball changes to

\[
p(B \mid H_2 \land H_3) = p(B \mid H_3) = 0
\]

Given the correspondence between the conditional independence intuitions (1) and the modularity property (3) just above, it follows that the rules describing this situation cannot be semantically modular. Indeed, using (2) we find that

\[
C(H_i \mid B, W) = C(H_i \mid B, H)
\]

and therefore

\[
C(H_i \mid B, W) = C(H_i \mid B, W)
\]

Intuitively, if a black ball is drawn first, one's belief in \( H_2 \) does not change significantly. However, if a white ball is drawn following the draw of a white ball, \( H_2 \) is ruled out and \( H_3 \) is increased. Thus, the certainty factor for \( H_2 \) depends on other pieces of evidence (either rules in the inference \( \Delta \)). Consequently, the rules representing this knowledge are not modular.

The lack of modularity can be traced directly to the fact that there are more than two mutually exclusive and exhaustive events. In such cases, \( H_i \) is a "mixture" of hypotheses and thus evidence will not be conditionally independent given \( \neg H \), even when evidence is conditionally independent given each \( H_i \). Since the conditional independence assumption (3) is not satisfied, the modularity property cannot hold. This result can be rigorously derived. It can be shown that whenever a set of mutually exclusive and exhaustive hypotheses contains more than two elements, the conditional independence assumption (3), and hence the modularity assumption, is incompatible with multiplicative update (1).
Figure 3. An influence diagram for the three urn problem

for this problem is shown in Figure 3. The two nodes labeled "Identity of urn" and "Color of ball drawn" in the upper level of the diagram represent the propositions relevant to the problem. The table in the lower level lists the possible values for each proposition. The arc between the two nodes in the upper level means that the probability distribution for "Color of ball drawn" depends on the value of "Identity of urn." Consequently, the probability distributions for "Color of ball drawn" given in the second level of the diagram is conditioned on each of the three possible values of "Identity of urn": $H_0$, $H_1$, and $H_2$. Note that since there are no arcs into the "Identity of urn" node, an unconditional or marginal distribution for this proposition is given. Also note that the same urn problem can be represented by an influence diagram with the arcs reversed. In this case, a marginal probability distribution would be assigned to "Color of ball drawn" and a conditional probability distribution would be assigned to "Identity of urn."

We see that there are several significant differences between influence diagrams and inference nets. The first difference is that an influence diagram is a two-level structure while the inference net contains only one level. Another difference is that propositions in an influence diagram can take on any number (possibly infinite) of mutually exclusive and exhaustive values. In an inference net, propositions typically can only take on the values "true" and "false." Another distinction is that influence diagrams express uncertain relationships among propositions using the concepts of probabilistic dependency while inference nets represent uncertain relationships using the concepts of belief update.

The story of Mr. Holmes illustrates another difference between influence diagrams and inference nets. The top level of an influence diagram for Mr. Holmes' situation is shown in Figure 4. Notice that many of the nodes in the graph are not directly connected by arcs. An intuitional view of this might be expressed as a lack of conditional independence. For example, the lack of a direct arc between "Burglar" and "Phone call" indicates "Burglar" influences "Phone call" only through its influence on "Alarm." In other words, "Burglar" and "Phone call" are conditionally independent given "Alarm." This would not be true if, for example, Mr. Holmes believed his neighbor might be the thief. We can thus see that this example of an influence diagram illustrates a principle by which experts can make point assumptions of conditional independence that concord with their beliefs. That is, assumptions of conditional independence are used by the methodology in semantically modular inference nets.

Due to differences between inference nets and influence diagrams, problems that can be

Figure 4. An influence diagram for Mr. Holmes' situation

represented in the former approach can be represented in the latter. For example, the three urn problem could not be represented using an inference net because there were three mutually exclusive and exhaustive hypotheses. However, representing more than two mutually exclusive and exhaustive hypotheses is straightforward in influence diagrams. The problem of Mr. Holmes could not be represented using an inference net because of strong dependencies among "Alarm," "Burglar," and "Earthquake." In an influence diagram, however, these dependencies are naturally represented by the two arcs entering the "Alarms." male. In particular, since there are arcs from both the burglar and earthquake propositions to the alarm proposition, the second level of the influence diagram will contain the probability distributions for "Alarm" as a function of all the possible values of both of these propositions. Thus, the following probabilities will be included in the lower level of the influence diagram:

\[
p(\text{Alarm} | \text{Burglar} | \text{Earthquake})\]
\[
p(\text{Alarm} | \text{Burglar} | \text{Earthquake})\]
\[
p(\text{Alarm} | \text{Burglar} | \text{Earthquake})\]
\[
p(\text{Alarm} | \text{Burglar} | \text{Earthquake})\]
\[
p(\text{Alarm} | \text{Burglar} | \text{Earthquake})\]
\[
p(\text{Alarm} | \text{Burglar} | \text{Earthquake})\]

The interactions between the burglar and earthquake hypotheses is completely captured by this probability distribution. In general, it can be shown that if an interference problem can be solved as a decision-theoretic framework then it can be represented with an influence diagram [10]. As we have seen, the same cannot be said for inference nets.

Now we are ready to consider a weaker notion of semantic modularity associated with the influence diagram representation. Imagine that a proposition is added to the influence diagram. When this occurs, the expert must first examine the dependency structure of the diagram. For example, the new node may be influenced by other nodes, may itself influence other nodes, or may introduce conditional independence or conditional dependence among nodes currently in the network. Thus, the expert must update the probability distributions for each node which has its incoming arcs modified. However, given the structure of an arc, in an influence diagram, there is no need to modify the probability distributions for the nodes in the network whose incoming arcs were not modified. Similarly, if a proposition is deleted from an influence diagram, the expert need only redefine dependencies in the network and then modify only the probability distributions for those nodes which had its incoming arcs

D.E. Heckerman and J.J. Horvitz

Phone call

Alarm

(Burglar)

Earthquake

Radio announcement

Figure 4. An influence diagram for Mr. Holmes' situation.